## Graphs and Genomes

## Michael Schatz

July 26, 2013
CSHL Undergraduate Research Program


## Outline

I. Graph Searching
2. Assembly by analogy
3. Genome Assembly


## Graphs



- Nodes
- People, Proteins, Genes, Neurons, Sequences, Numbers, ...
- Edges
- $A$ is connected to $B$
- $A$ is related to $B$
- A regulates $B$
- A precedes B
- A interacts with $B$
- A activates B
- ...


## Graph Types



## Representing Graphs



| Adjacency Matrix <br> Good for dense graphs <br> Fast, Fixed storage: N bits |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | D | E | F | C |  |
| A |  |  | I | I | I |  |  |  |
| B |  |  |  | I | I |  |  |  |
| C |  |  |  |  |  | I | I |  |
| D |  |  |  |  |  | I |  |  |
| E |  |  |  |  |  |  | I |  |
| F |  |  |  |  |  |  |  |  |
| G |  |  |  |  |  |  | I |  |

Adjacency Matrix
Good for dense graphs
Fast, Fixed storage: $\mathrm{N}^{2}$ bits

## Adjacency List

Good for sparse graphs
Compact storage: 4 bytes/edge
A: C, D, E
D: F
B: D, E
E: F
C: F, G
G:

Edge List
Easy, good if you (mostly) need to iterate through the edges 8 bytes / edge

| A,C | B,C |  | C,F |  |
| :---: | :---: | :---: | :---: | :---: |
| A,D | B,D |  | C,G |  |
| A,E | B,E |  | D,F |  |
|  | E,F | F,G |  |  |

Tools
Matlab: http://www.mathworks.com/
Graphviz: http://www.graphviz.org/ Gephi: https://gephi.org/
Cytoscape: http://www.cytoscape.org/
digraph G \{
A->B
$B->C$
A->C
\}
dot -Tpdf -og.pdf g.dot


## Network Characteristics

|  | C. elegans | D. melanogaster | S. cerevisiae |
| :---: | :---: | :---: | :---: |
| \# Nodes | 2646 | 7464 | 4965 |
| \# Edges | 4037 | 22831 | 17536 |
| Avg. / Max Degree | 3.0 / 187 | 6.1 / 178 | 7.0 / 283 |
| \# Components | 109 | 66 | 32 |
| Largest Component | 2386 | 7335 | 4906 |
| Diameter | 14 | 12 | 11 |
| Avg. Shortest Path | 4.8 | 4.4 | 4.1 |
| Data Sources | 2H | 2x2H, TAP-MS | $8 \times 2 \mathrm{H}, 2 \times \mathrm{TAP}, \mathrm{SUS}$ |
| Degree <br> Distributions |  |  |  |

Small World: Avg. Shortest Path between nodes is small
Scale Free: Power law distribution of degree - preferential attachment

## Network Motifs

- Network Motif
- Simple graph of connections
- Exhaustively enumerate all possible I, 2, 3, ... k node motifs
- Statistical Significance
- Compare frequency of a particular network motif in a real network as compared to a randomized network
- Certain motifs are "characteristic features" of the network

| Network | Nodes | Edges | $N_{\text {real }} N_{\text {rand }} \pm$ SD | $Z$ score | $N_{\text {real }} \quad N_{\text {rand }} \pm$ SD | Z score | $N_{\text {real }} \quad N_{\text {rand }} \pm$ SD | $Z$ score |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Gene regulation (transcription) |  |  |  | Feedforward loop | ${\underset{z}{z}}_{x}^{x}$ | Bi-fan |  |  |
| E. coli <br> S. cerevisiae* | $\begin{aligned} & 424 \\ & 685 \\ & \hline \end{aligned}$ | $\begin{array}{r} 519 \\ 1,052 \\ \hline \end{array}$ | $\begin{array}{rr} 40 & 7 \pm 3 \\ 70 & 11 \pm 4 \\ \hline \end{array}$ | $\begin{aligned} & 10 \\ & 14 \end{aligned}$ | $\begin{array}{rr} 203 & 47 \pm 12 \\ 1812 & 300 \pm 40 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ 41 \\ \hline \end{array}$ |  |  |
| Neurons |  |  | $\left[\begin{array}{l} \mathrm{X} \\ \mathrm{~V} \\ \mathrm{Y} \\ \mathrm{~V} \\ \mathrm{Z} \end{array}\right.$ | Feedforward loop |  | Bi-fan | $\begin{aligned} & k_{w}^{x} v^{y} \\ & v^{z} \end{aligned}$ | Biparallel |
| C. elegans $\dagger$ | 252 | 509 | $125 \quad 90 \pm 10$ | 3.7 | $127 \quad 55 \pm 13$ | 5.3 | $227 \quad 35 \pm 10$ | 20 |
| Food webs |  |  | $\begin{aligned} & \hline \mathbf{X} \\ & \mathbf{V} \\ & \mathbf{Y} \\ & \mathbf{V} \\ & \mathbf{z} \end{aligned}$ | Three chain | $\begin{gathered} k_{w}^{x} v \\ Y_{V}^{z} \end{gathered}$ | Biparallel |  |  |
| Little Rock | 92 | 984 | $3219 \quad 3120 \pm 50$ | 2.1 | $7295 \quad 2220 \pm 210$ | 25 |  |  |
| Ythan | 83 | 391 | $1182 \quad 1020 \pm 20$ | 7.2 | $1357 \quad 230 \pm 50$ | 23 |  |  |
| St. Martin | 42 | 205 | $469 \quad 450 \pm 10$ | NS | $382 \quad 130 \pm 20$ | 12 |  |  |
| Chesapeake | 31 | 67 | $80 \quad 82 \pm 4$ | NS | $26 \quad 5 \pm 2$ | 8 |  |  |
| Coachella | 29 | 243 | $279 \quad 235 \pm 12$ | 3.6 | $181 \quad 80 \pm 20$ | 5 |  |  |
| Skipwith | 25 | 189 | $184 \quad 150 \pm 7$ | 5.5 | $397 \quad 80 \pm 25$ | 13 |  |  |
| B. Brook | 25 | 104 | $181 \quad 130 \pm 7$ | 7.4 | $267 \quad 30 \pm 7$ | 32 |  |  |
| Electronic circuits (forward logic chips) |  |  | $\left[\begin{array}{l}\text { X } \\ \underset{\sim}{\mathrm{V}} \\ \stackrel{\rightharpoonup}{V} \\ > \\ \mathrm{Z}\end{array}\right.$ | Feed- <br> forward <br> loop | $V_{z}^{x}$ | Bi-fan |  | Biparallel |
| s15850 | 10,383 | 14,240 | $424 \quad 2 \pm 2$ | 285 | $1040 \quad 1 \pm 1$ | 1200 | $480 \quad 2 \pm 1$ | 335 |
| s38584 | 20,717 | 34,204 | $413 \quad 10 \pm 3$ | 120 | $1739 \quad 6 \pm 2$ | 800 | $711 \quad 9 \pm 2$ | 320 |
| s38417 | 23,843 | 33,661 | $6123 \pm 2$ | 400 | 2404 1 $\pm 1$ | 2550 | $531 \quad 2 \pm 2$ | 340 |
| s9234 | 5,844 | 8,197 | $211 \quad 2 \pm 1$ | 140 | $7541 \pm 1$ | 1050 | $2091 \pm 1$ | 200 |
| s13207 | 8,651 | 11,831 | $403 \quad 2 \pm 1$ | 225 | $4445 \quad 1 \pm 1$ | 4950 | $264 \quad 2 \pm 1$ | 200 |
| Electronic circuits(digital fractional multipliers) |  |  | $\underset{\mathrm{x} \longleftarrow}{\prod_{\mathrm{z}}^{\mathrm{x}} \searrow}$ | Three node feedback loop |  | Bi-fan |  | Fournode feedback loop |
| s208 | 122 | 189 | $10 \quad 1 \pm 1$ | 9 | $4 \quad 1 \pm 1$ | 3.8 | $5 \quad 1 \pm 1$ | , |
| s420 | 252 | 399 | $20 \quad 1 \pm 1$ | 18 | $10 \quad 1 \pm 1$ | 10 | $11 \quad 1 \pm 1$ | 11 |
| 8838 $\ddagger$ | 512 | 819 | $40 \quad 1 \pm 1$ | 38 | $22 \quad 1 \pm 1$ | 20 | $23 \quad 1 \pm 1$ | 25 |
| World Wide Web |  |  | $\left[\begin{array}{l} x \\ \hat{\lambda} \\ \vdots \\ \vdots \\ z \end{array}\right.$ | Feedback <br> with two <br> mutual <br> dyads |  | Fully connected triad |  | Uplinked mutual dyad |
| nd.edu§ | 325,729 | 1.46e6 | $1.1 \mathrm{e} 5 \quad 2 \mathrm{e} 3 \pm 1 \mathrm{e} 2$ | 800 | $6.8 \mathrm{e} 6 \quad 5 \mathrm{e} 4 \pm 4 \mathrm{e} 2$ | 15,000 | $1.2 \mathrm{e} 6 \quad 1 \mathrm{e} 4 \pm 2 \mathrm{e} 2$ | 5000 |

Network Motifs: Simple Building Blocks of Complex Networks
Milo et al (2002) Science. 298:824-827

## Modularity

- Community structure
- Densely connected groups of vertices, with only sparser connections between groups
- Reveals the structure of large-scale network data sets
- Modularity
- The number of edges falling within groups minus the expected number in an equivalent network with edges placed at random
- Larger positive values => Stronger community structure
- Optimal assignment determined by computing the eigenvector of the modularity matrix
Modularity and community structure in networks.
Newman ME (2006) PNAS. I03(23) 8577-8582

$$
\begin{aligned}
& \qquad Q=\frac{1}{4 m} \sum_{i j} \underbrace{\left(A_{i j}-\right.}_{\uparrow}-\frac{k_{i} k_{j}}{2 m}) \\
& \begin{array}{c}
\uparrow \\
\begin{array}{c}
\text { Normalization } \\
\text { factor }
\end{array} \\
\begin{array}{c}
\text { Adjacency } \\
\text { matrix }
\end{array} \\
\begin{array}{c}
\text { Indicates } \\
\text { same group }
\end{array}
\end{array}
\end{aligned}
$$

Random Prob. (product of degrees)

## Kevin Bacon and Bipartite Graphs

Find the shortest path from
Kevin Bacon
to
Jason Lee

Breadth First Search:
4 hops
Bacon Distance:
2


## BFS and TSP

- BFS computes the shortest path between a pair of nodes in $\mathrm{O}(|\mathrm{E}|)=\mathrm{O}\left(|\mathrm{N}|^{2}\right)$
- What if we wanted to compute the shortest path visiting every node once?
- Traveling Salesman Problem

$$
\begin{aligned}
& \text { ABDCA: } 4+2+5+3=14 \\
& \text { ACDBA: } 3+5+2+4=14^{*} \\
& \text { ABCDA: } 4+1+5+1=11 \\
& \text { ADCBA: } 1+5+1+4=11 * \\
& \text { ACBDA: } 3+1+2+1=7 \\
& \text { ADBCA: } 1+2+1+3=7 *
\end{aligned}
$$



## Greedy Search



## Greedy Search

## Greedy Search

cur=graph.randNode()
while (!done)


Greedy: $\quad$ ABDCA $=5+8+10+50=73$
Optimal: $A C B D A=5+11+10+12=38$

Greedy finds the global optimum only when
I. Greedy Choice: Local is correct without reconsideration
2. Optimal Substructure: Problem can be split into subproblems

Optimal Greedy: Making change with the fewest number of coins

## TSP Complexity

- No fast solution
- Knowing optimal tour through n cities doesn't seem to help much for $n+1$ cities
[How many possible tours for n cities?]

- Extensive searching is the only provably correct algorithm
- Brute Force: O(n!)
- $\sim 20$ cities max
- $20!=2.4 \times 10^{18}$
- Branch-and-Bound can often help



## TSP and NP-complete

- TSP is one of many extremely hard problems of the class NP-complete
- Extensive searching is the only way to find an exact solution
- Often have to settle for approx. solution

- WARNING: Many biological problems are in this class
- Find a tour the visits every node once (Genome Assembly)
- Find the smallest set of vertices covering the edges (Essential Genes)
- Find the largest clique in the graph (Protein Complexes)
- Find the highest mutual information encoding scheme (Neurobiology)
- Find the best set of moves in tetris
- ...
- http://en.wikipedia.org/wiki/List_of_NP-complete_problems



## Outline

I. Graph Searching
2. Assembly by analogy
3. Genome Assembly

## Shredded Book Reconstruction

- Dickens accidentally shreds the first printing of A Tale of Two Cities
- Text printed on 5 long spools




| It was | t thessbldse be.simoestinite | wass and lweonstrof tintesses | it was the age of |  |
| :---: | :---: | :---: | :---: | :---: |



- How can he reconstruct the text?
- 5 copies $\times 138,656$ words $/ 5$ words per fragment $=138 \mathrm{k}$ fragments
- The short fragments from every copy are mixed together
- Some fragments are identical

It was the best of
age of wisdom, it was
best of times, it was
it was the age of
it was the age of
it was the worst of
of times, it was the
of times, it was the
of wisdom, it was the
the age of wisdom, it
the best of times, it
the worst of times, it
times, it was the age
times, it was the worst
was the age of wisdom,
was the age of foolishness,
was the best of times,
was the worst of times,
wisdom, it was the age
worst of times, it was

## Greedy Reconstruction

```
It was the best of
|\mp@code{was the best of times,}
times, it was the worst
times, it was the age
```

The repeated sequence make the correct reconstruction ambiguous

- It was the best of times, it was the [worst/age]

Model sequence reconstruction as a graph problem.

## de Bruijn Graph Construction

- $\mathrm{D}_{\mathrm{k}}=(\mathrm{V}, \mathrm{E})$
- $V=$ All length- $k$ subfragments $(k<l)$
- $E=$ Directed edges between consecutive subfragments
- Nodes overlap by k-I words

Original Fragment

It was the best of

Directed Edge

- Locally constructed graph reveals the global sequence structure
- Overlaps between sequences implicitly computed
de Bruijn, 1946
Idury and Waterman, 1995
Pevzner, Tang, Waterman, 2001


## de Bruijn Graph Assembly



## de Bruijn Graph Assembly




## Outline

I. Genome Assembly by Analogy
2. Graph Searching
3. Genome Assembly

## Milestones in Genome Assembly


1977. Sanger et al. ${ }^{\text {st }}$ Complete Organism 5375 bp

2000. Myers et al.
$\|^{\text {st }}$ Large WGS Assembly.
Celera Assembler. I 16 Mbp

1995. Fleischmann et al.
$\|^{\text {st }}$ Free Living Organism TIGR Assembler. I.8Mbp


200I.Venter et al., IHGSC Human Genome
Celera Assembler/GigaAssembler. 2.9 Gbp

1998. C. elegans SC ${ }^{\text {st }}$ Multicellular Organism BAC-by-BAC Phrap. 97Mbp

2010. Li et al.
$\|^{\text {st }}$ Large SGS Assembly.
SOAPdenovo 2.2 Gbp

Like Dickens, we must computationally reconstruct a genome from short fragments

## Current Applications

- Novel genomes

- Metagenomes

- Sequencing assays
- Structural variations
- Transcript assembly



## Assembling a Genome

I. Shear \& Sequence DNA

2. Construct assembly graph from overlapping reads

GGATGCGCGACACGTCGCATATCCGGT...
3. Simplify assembly graph

4. Detangle graph with long reads, mates, and other links


## Illumina Sequencing by Synthesis



1. Prepare
2. Attach


3. Image

4. Basecall

Metzker (20I0) Nature Reviews Genetics I I:3I-46

## Typical contig coverage



Imagine raindrops on a sidewalk

## Balls in Bins Ix



Balls in Bins
balls in bin
Total balls: $\mathbf{1 0 0 0}$


## Balls in Bins $2 x$



Balls in Bins
balls in bin
Total balls: 2000


## Balls in Bins 3x



Balls in Bins
Total balls: $\mathbf{3 0 0 0}$


## Balls in Bins 4x



Balls in Bins
Total balls: $\mathbf{4 0 0 0}$


## Balls in Bins 5x



Balls in Bins
Total balls: 5000


## Balls in Bins 6x



Balls in Bins
Total balls: $\mathbf{6 0 0 0}$


## Balls in Bins 7x



Balls in Bins
Total balls: 7000


## Balls in Bins $8 x$



Balls in Bins
Total balls: $\mathbf{8 0 0 0}$


## Genome Coverage Distribution



Expect Poisson distribution on depth
Standard Deviation $=\operatorname{sqrt}(\mathrm{cov})$
This is the mathematically model => reality may be much worse
Double your coverage for diploid genomes

## Initial Contigs

- After simplification and correction, compress graph down to its non-branching initial contigs
- Aka "unitigs","unipaths"



## Repeats and Read Length



- Explore the relationship between read length and contig N50 size
- Idealized assembly of read lengths: 25, 35,50, I00, 250, 500, 1000
- Contig/Read length relationship depends on specific repeat composition

Assembly Complexity of Prokaryotic Genomes using Short Reads. Kingsford C, Schatz MC, Pop M (20I0) BMC Bioinformatics. II:2I.

## Repetitive regions

- Over $50 \%$ of the human genome is repetitive

| Repeat Type | Definition / Example | Prevalence |
| :---: | :---: | :---: |
| Low-complexity DNA / Microsatellites | $\left(b_{1} b_{2} \ldots b_{k}\right)^{N}$ where $I \leq k \leq 6$ CACACACACACACACACACA | 2\% |
| SINEs (Short Interspersed Nuclear Elements) | Alu sequence ( $\sim 280 \mathrm{bp}$ ) <br> Mariner elements ( $\sim 80 \mathrm{bp}$ ) | 13\% |
| LINEs (Long Interspersed Nuclear Elements) | $\sim 500-5,000 \mathrm{bp}$ | 21\% |
| LTR (long terminal repeat) retrotransposons | Tyl-copia, Ty3-gypsy, Pao-BEL (~100-5,000 bp) | 8\% |
| Other DNA transposons |  | 3\% |
| Gene families \& segmental duplications |  | 4\% |

## Repeats and Coverage Statistics



- If $n$ reads are a uniform random sample of the genome of length $G$, we expect $k=n \Delta / G$ reads to start in a region of length $\Delta$.
- If we see many more reads than $k$ (if the arrival rate is $>A$ ), it is likely to be a collapsed repeat
- Requires an accurate genome size estimate
$\operatorname{Pr}(X-$ copy $)=\binom{n}{k}\left(\frac{X \Delta}{G}\right)^{k}\left(\frac{G-X \Delta}{G}\right)^{n-k}$

$$
A(\Delta, k)=\ln \left(\frac{\operatorname{Pr}(1-\text { copy })}{\operatorname{Pr}(2-\text { copy })}\right)=\ln \left(\frac{\frac{(\Delta n / G)^{k}}{k!} e^{\frac{-\Delta n}{G}}}{\frac{(2 \Delta n / G)^{k}}{k!} e^{\frac{-2 \Delta n}{G}}}\right)=\frac{n \Delta}{G}-k \ln 2
$$

## Scaffolding

- Initial contigs (aka unipaths, unitigs) terminate at
- Coverage gaps: especially extreme GC regions
- Conflicts: sequencing errors, repeat boundaries
- Iteratively resolve longest, 'most unique' contigs
- Both overlap graph and de Bruijn assemblers initially collapse repeats into single copies
- Uniqueness measured by a statistical test on coverage



## N50 size

Def: $50 \%$ of the genome is in contigs larger than N50

Example: I Mbp genome 50\%


N50 size $=30 \mathrm{kbp}$
$(300 k+100 k+45 k+45 k+30 k=520 k>=500 k b p)$
Note:
N50 values are only meaningful to compare when base genome size is the same in all cases


## Assembly Algorithms

| ALLPATHS-LG | SOAPdenovo | Celera Assembler |
| :---: | :---: | :---: |
| Broad's assembler <br> (Gnerre et al. 20II) |  <br> BGI's assembler (Li et al. 20IO) |  <br> JCVI's assembler (Miller et al. 2008) |
| De bruijn graph Short + PacBio (patching) | De bruijn graph Short reads | Overlap graph <br> Medium + Long reads |
| Easy to run if you have compatible libraries | Most flexible, but requires a lot of tuning | Supports Illumina/454/PacBio Hybrid assemblies |
| http://www.broadinstitute.org/ software/allpaths-Ig/blog/ | http://soap.genomics.org.cn/ soapdenovo.htm | http://wgs-assembler.sf.net |

## PacBio Error Correction \& Assembly http://wgs-assembler.sf.net

I. Correction Pipeline
I. Map short reads (SR) to long reads (LR)
2. Trim LRs at coverage gaps
3. Compute consensus for each LR

2. Error corrected reads can be easily assembled, aligned


Hybrid error correction and de novo assembly of single-molecule sequencing reads. Koren, S, Schatz, MC, et al (2012) Nature Biotechnology. doi:IO.I038/nbt. 2280

## Scalpel: Haplotype Microassembly

G. Narzisi, D. Levy, I. lossifov, J. Kendall, M.Wigler, M. Schatz

- Use assembly techniques to identify complex variations from short reads
- Improved power to find indels
- Trace candidate haplotypes sequences as paths through assembly graphs

Ref: ...TCAGAACAGCTGGATGAGATCTTAGCCAACTACCAGGAGATTGTCTTTGCCCGGA. . .

Father: ...TCAGAACAGCTGGATGAGATCTTAGCCAACTACCAGGAGATTGTCTTTGCCCGGA...
Mother: . . .TCAGAACAGCTGGATGAGATCTTAGCCAACTACCAGGAGATTGTCTTTGCCCGGA. . .
Sib: ...TCAGAACAGCTGGATGAGATCTTAGCCAACTACCAGGAGATTGTCTTTGCCCGGA...
Aut(1): ...TCAGAACAGCTGGATGAGATCTTAGCCAACTACCAGGAGATTGTCTTTGCCCGGA...
Aut(2): ...TCAGAACAGCTGGATGAGATCTTACC------CCGGGAGATTGTCTTTGCCCGGA. . .
6bp heterozygous indel at chr I3:25280526 ATP I2A

## Assembly Summary

Graphs are ubiquitous in the world

- Pairwise searching is easy, finding features is hard

Assembly quality depends on
I. Coverage: low coverage is mathematically hopeless
2. Repeat composition: high repeat content is challenging
3. Read length: longer reads help resolve repeats
4. Error rate: errors reduce coverage, obscure true overlaps

Assembly is a hierarchical, starting from individual reads, build high confidence contigs/unitigs, incorporate the mates to build scaffolds

- Extensive error correction is the key to getting the best assembly possible from a given data set


## Genomics Challenges

The foundations of genomics will continue to be observation, experimentation, and interpretation

- Technology will continue to push the frontier
- Measurements will be made digitally over large populations, at extremely high resolution, and for diverse applications


## Rise in Quantitative and Computational Demands

I. Experimental design: selection, collection \& metadata
2. Observation: measurement, storage, transfer, computation
3. Integration: multiple samples, assays, analyses
4. Discovery: visualizing, interpreting, modeling

Ultimately limited by the human capacity to execute extremely complex experiments and interpret results

## Acknowledgements

Schatz Lab
Giuseppe Narzisi
Shoshana Marcus
James Gurtowski
Alejandro Wences
Hayan Lee
Rob Aboukhalil
Mitch Bekritsky
Charles Underwood
Rushil Gupta
Avijit Gupta
Shishir Horane
Deepak Nettem
Varrun Ramani
Piyush Kansal
Greg Vurture
Aspyn Palatnick

CSHL Hannon Lab
Gingeras Lab
Iossifov Lab
Levy Lab
Lippman Lab
Lyon Lab
Martienssen Lab
McCombie Lab
Ware Lab
Wigler Lab

IT Department

NBACC
Adam Phillippy
Sergey Koren

## SFARI

SIMONS FOUNDATION AUTISM RESEARCH INITIATIVE

U.S. DEPARTMENT OF ENERGY


## Thank You


http://schatzlab.cshl.edu/teaching/ @mike_schatz

